



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

THE AMERICAN MATHEMATICAL MONTHLY.

Entered at the Post-office at Springfield, Missouri, as Second-class Mail Matter.

VOL. III.

MARCH, 1896.

No. 3.

NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

By BENJ. F. YANNEY, A. M., Mount Union College, Alliance, Ohio, and JAMES A. CALDERHEAD, B. Sc.,
Superintendent of Schools, Limaville, Ohio.

It is proposed in these papers to give a more or less complete list of proofs, both new and old, of this celebrated and practical theorem. An attempt is made at classification based upon immediate principles used in the proofs. Due credit will be given in all known cases. A historical note will be appended to the completed list.

THEOREM.

The square described upon the hypotenuse of a right triangle is equivalent to the sum of the squares described upon the other two sides.

PROOFS.

I. RESULTING FROM LINEAR RELATIONS OF SIMILAR RIGHT TRIANGLES.

Let ABC be a $\triangle$ right-angled at C . Draw CD perpendicular to AB . There are thus three similar right triangles.

Letting $AC=b$, $BC=a$, $AB=c$, $CD=x$, $AD=y$, $BD=c-y$, we obtain the following proportions, with their resulting equations:

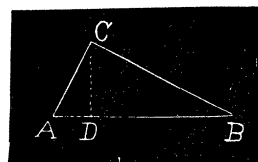


Fig. 1.

- (1). $y : b :: b : c$. $\therefore yc=b^2$ 1.
- (2). $y : b :: x : a$. $\therefore bx=ay$ 2.
- (3). $b : c :: x : a$. $\therefore cx=ab$ 3.
- (4). $y : x :: x : c-y$. $\therefore x^2=cy-y^2$ 4

- (5). $y : x :: b : a. \therefore bx=ay$ 2.
 (6). $x : c-y :: b : a. \therefore ax=b(c-y)$ 5.
 (7). $c-y : a :: a : c. \therefore c(c-y)=a^2$ 6.
 (8). $c-y : a :: x : b. \therefore ax=b(c-y)$ 5.
 (9). $a : c :: x : b. \therefore cx=ab$ 3.

From the nine different proportions, there are derived but six different equations, equation 2 being derived from proportion (2) or (5), 3 from (3) or (9), and 5 from (6) or (8).

It is evident that from no single equation can we determine the relation between a , b , and c , the sides of the given right $\triangle$.

It is also evident that there is but one set of twos which will give the relation desired, viz., equations 1 and 6. If we add these, member by member, we get directly $c^2=a^2+b^2$. Giving to this form the usual geometrical interpretation, we thus have one proof of the theorem. This, though in a different form, is one of the methods usually found in the books. It is credited to Legendre.

We now proceed to find combinations of threes, which will give the required relation. There are $\frac{6.5.4}{1.3}=20$ sets of three equations out of the six. But of

these, four must be rejected, since they contain 1 and 6, which two alone prove the theorem, as already shown; also the following three sets, since in each set the equations are dependent: 1, 2, 3; 2, 4, 5; 3, 5, 6. There are, then, left the following thirteen sets, from each of which, if we eliminate x and y , we get $c^2=a^2+b^2$: 1, 2, 4; 1, 2, 5; 1, 3, 4; 1, 3, 5; 1, 4, 5; 2, 3, 4; 2, 3, 5; 2, 3, 6; 2, 4, 6; 2, 5, 6; 3, 4, 5; 3, 4, 6; 4, 5, 6.

Of these thirteen sets, there are six that contain one equation each, derived from either of two proportions; six sets containing two each such equations; and one containing three. Therefore, including the proof already given, there are $1+6 \times 2+6 \times 2^2+2^3=45$ proofs, by this method.

II. Let ABC be a $\triangle$ right-angled at C . Draw a line perpendicular to AB from A , meeting BC produced, as at D .

Letting $AC=b$, $BC=a$, $AB=c$, $AD=x$, $DC=y$, $BD=y+a$, and proceeding as in the preceding case, we find that this method also yields 45 different proofs. The details are left to be carried out by the reader.

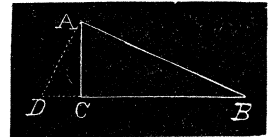


Fig. 2.

III. Let ABC be a $\triangle$ right-angled at C . Draw DE perpendicular to AB so that $DE=DC$. Then will $BE=BC$. $\triangle ADE$ is similar to $\triangle ABC$.

Letting $AC=b$, $BC=a$, $AB=c$, $AE=c-a$, $DE=DC=x$, $AD=b-x$, we have :

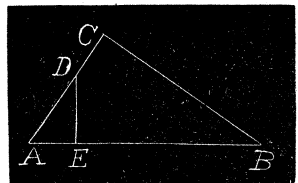


Fig. 3.

$$(1). \quad c-a : b :: x : a. \quad \therefore x = \frac{ac-a^2}{b} \dots\dots\dots 1.$$

$$(2). \quad c-a : b :: b-x : c. \quad \therefore x = \frac{b^2-c^2+ac}{b} \dots\dots\dots 2.$$

$$(3). \quad x : a :: b-x : c. \quad \therefore x = \frac{ab}{a+c} \dots\dots\dots 3.$$

From the three equations, it is evident that we may obtain three proofs by this method.

IV. Let ABC be a $\triangle$ right-angled at C . Extend AB to D making $BD=BC$. Draw a line perpendicular to AD at D , meeting AC produced as at E . Then will $CE=DE$, and $\triangle AED$ will be similar to $\triangle ABC$.

Letting $AC=b$, $BC=a$, $AB=c$, $AD=c+a$, $DE=x$, $AE=x+b$, and proceeding as in the last case, we obtain three more proofs, making in all, thus far, 96 proofs.

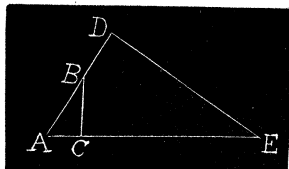


Fig. 4.

In our next paper, we shall give a method whose results reach into the thousands.

[To be Continued.]

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from February Number.]

PROPOSITION XXIII. *If any two straight lines AX , BX (Fig. 27.) exist in the same plane, either they have (even in the hypothesis of acute angle) a common perpendicular, or prolonged toward either the same part, unless sometime at a finite distance one strikes upon the other, they mutually approach ever more toward each other.*

Proof. From any point A of AX is let fall to the straight BX the perpendicular AB . If BA makes with AX a right angle, we have the asserted case of the common perpendicular. But otherwise this straight makes toward one or the other part, as suppose toward the parts of the point X , an acute angle.